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BSc6MatC601x **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2023 (NEW COURSE)
Graph Theory and Complex Analysis-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following question. (Any Two) (10)

1. Prove that a simple graph with $\boldsymbol{n}$ ***vertices and*** $\boldsymbol{k}$ ***components*** can have atmost $\frac{1}{2}(\boldsymbol{n}-\boldsymbol{k})(\boldsymbol{n}-\boldsymbol{k}+\boldsymbol{1})$ number of edges.
2. Define: Euler Graph
   Prove that a connected graph G is an Euler graph ***if and only if*** it can be decomposed into circuits.
3. Define: Spanning Tree
   Prove that a connected graph G with $\boldsymbol{n}$ ***vertices*** & $\boldsymbol{e}$ ***edges*** is having exactly $\boldsymbol{n}-\boldsymbol{1}$ tree branches and $\boldsymbol{e}-\boldsymbol{n}+\boldsymbol{1}$ chords with respect to any spanning tree.

Q.1(B) Answer the following questions. (Any One) (04)

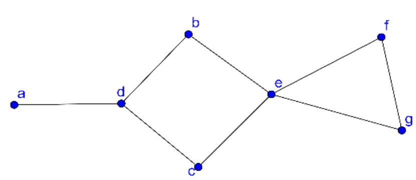
1. Prove that a graph G is a tree graph ***if and only if*** there is exactly one path between every pair of vertices.
2. Explain graph operations **Ring Sum** and **Decomposition** by taking suitable graphs.

Q.2(A) Answer the following question. (Any Two) (10)

1. Define: Planar Graph
   Prove that Kuratowski's first graph is non-planar.
2. State and prove the Euler's formula for connected planar graphs.
3. Define: Incidence matrix.
   Prove that the rank of the incidence matrix $\boldsymbol{A}(\boldsymbol{G})$ of a connected graph $\boldsymbol{G}$ with $\boldsymbol{n}$ vertices is $\boldsymbol{n}-\boldsymbol{1}$.

Q.2(B) Answer the following questions. (Any One) (04)

1. Define: Self Dual Graph
   Prove that $\boldsymbol{K_4}$ is a self dual graph.
2. By applying the laws of Boolean Algebra, find all maximal independent sets of the following graph.



Q.3(A) Answer the following question. (Any Two) **(10)**

1. Prove that the set of all bilinear maps forms a group w.r.t. the binary operation of composition of mappings.
2. Prove that the Mobious Transformation maps circles or straight lines into circles or straight lines.
3. Prove that the mapping $\boldsymbol{w = z^2}$ is a Conformal Mapping.

Q.3(B) Answer the following questions. (Any One) **(04)**

1. Discuss the transformation $\boldsymbol{w = sinh(z)}$.
2. Prove that the transformation $\boldsymbol{w = z^2 + 2z}$ maps the unit circle of the $\boldsymbol{Z}$ −plane into a cardioid in the $\boldsymbol{W}$ −plane.

Q.4(A) Answer the following question. (Any Two) **(10)**

1. If the complex function $f(z)$ is analytic every where inside and on circle C_0 having radius r_0 and centered at z_0 and z is any point inside the circle C_0, then prove that, $\boldsymbol{f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \frac{f''(z_0)}{2!}(z - z_0)^2 + \frac{f'''(z_0)}{3!}(z - z_0)^3} + \cdots$.
2. State and prove Laurent's infinite series of an analytic function $\boldsymbol{f(z)}$.
3. Show that when $\boldsymbol{0 < |z| < 4}$,

$$\boldsymbol{\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{(n+2)}}}$$

Q.4(B) Answer the following questions. (Any One) **(04)**

1. Expand $\frac{1}{(z-1)(z-2)}$ in Laurentz's series for $1 < |z| < 2$.
2. Prove that $\boldsymbol{\frac{sinhz}{z^2} = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{z^{2n-1}}{(2n+1)!}}$.

Q.5(A) Answer the following question. (Any Two) **(10)**

1. Define: (i) Simple Pole (ii) Pole of order m
   If $\boldsymbol{z_0}$ is the pole of order $\boldsymbol{m(m \geq 2)}$ of a complex function $\boldsymbol{f(z)}$, then prove that $\boldsymbol{Res(f(z), z_0) = \frac{\varphi^{m-1}(z_0)}{(m-1)!}}$ **,** $\boldsymbol{where\ \varphi(z) = (z - z_0)^m f(z)}$ **.**
2. State and prove the Cauchy's residue theorem.
3. Prove: $\boldsymbol{\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3} = \frac{3\pi}{8}}$

Q.5(B) Answer the following questions. (Any One) **(04)**

1. Evaluate: $\boldsymbol{\int_0^{\infty} \frac{cosax}{x^2+1} dx, a} > 0$.
2. Evaluate: $\boldsymbol{\int_0^{2\pi} \frac{cos(3\theta)}{(5-4cos\theta)} d\theta}$.

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